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Evidence for change in the characteristics of strong interactions at ultra-high energies

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Abstract. In this paper an attempt has been made to understand the experimental results on the size dependence of several properties of high energy hadrons reported in a previous paper in terms of the characteristics of high energy collisions and primary composition using detailed Monte Carlo simulations. It is shown that the behaviour of the high energy hadron component, especially the size dependence, cannot be accounted for in terms of a change of primary composition alone in the energy range 10^{14}–10^{16} eV which has been suggested by some of the earlier investigations to explain certain properties of air showers. The present analysis strongly suggests the need for invoking rather drastic changes in the characteristics of strong interactions, at ultra-high energies. The details about the trends in the changes will be discussed in a subsequent paper.

1. Introduction

In the preceding paper (Vatcha and Sreekantan 1973, to be referred to as I) we have presented experimental results on changes in the properties of hadrons of energy greater than 25 GeV in air showers of size 5×10^4 to 3×10^6 particles at 800 g cm^{-2}, obtained by operating a 2 m^2 multiplate cloud chamber at the centre of the TIFR air shower array at Ooty and have compared the results with those of other experiments. In this paper we make a comparison of the experimental results with expectations based on existing Monte Carlo simulations of air showers in which the parameters of high energy collisions are adopted as reasonable extrapolations of measurements and trends at machine energies ($\leqslant 70$ GeV). The properties of heavy primary showers are obtained by a simple superposition of proton showers of the requisite number and energy per nucleon. The comparison shows a wide disparity between the experimental and calculated results. We find it difficult to explain in a consistent manner the size dependence of the properties of hadrons by any type of change in primary composition in the relevant energy range 10^{14}–10^{16} eV. While the variation with shower size of some of the. properties of the high energy hadrons demands a drastic transformation of the primary composition from a lighter to a heavier component within this energy range, certain other properties require just the opposite trend, suggesting thereby that the discrepancies cannot be removed by composition change alone. Besides there are several properties of hadrons which clearly show that the parameters of strong interactions like inelasticity and/or interaction mean free path and composition of secondary particles at ultra-high energies have to be necessarily different from those determined at machine energies.

2. Comparison of experimental results with Monte Carlo simulations

In recent years a number of investigators have carried out Monte Carlo simulations of the different components of air showers. We have chosen to compare our experimental results with the calculations of Murthy *et al* (1968) since in these calculations the results on the various parameters of interest are available for the specific altitude at which the present experiment has been carried out, that is 800 g cm^{-2}, and also in a form suitable for direct comparison. Most of the later simulations (see, for example, Greider 1970 and references therein) agree well with the calculations of Murthy *et al.*

In table 1, the details of the eight models, on the basis of which the hadronic cascades have been simulated by Murthy *et al*, are given. It is seen that Murthy *et al* have considered three different laws of multiplicity, the formation of isobars and also the formation of nucleon–antinucleon pairs. The simulations have been carried out both for proton and heavy primary induced showers.

In table 2, the experimental results on the variation of the slope of the hadron spectrum in the energy range 50–800 GeV with shower size is compared with the calculated slopes of Murthy *et al.* According to the experiment, the slope changes from about 1·3 or more at about 10^5 to approximately 1·9 or more at about 4×10^5 and thereafter remains constant. The calculated value however is 1·2 for proton and iron showers at all sizes and for all the models except the HLN model in which the value is 1·5. It is important to note that the calculations do not give any change of slope with size.

In table 3, the absolute number of hadrons of energy greater than 100 GeV in showers of different sizes obtained in the Ooty experiment are compared with the calculated values for both proton and iron induced showers. It is seen that the observed numbers are less by a factor of four to ten if we consider proton showers and models in which $N\bar{N}$ production is not considered. The disparity is enhanced by a further factor of two if $N\bar{N}$ production is considered which, as we shall see, is quite essential to explain various results. There is evidence for increase in the cross section for the production of $N\bar{N}$'s in the time structure experiments of Tonwar and Sreekantan (1971) and also in the ISR experiments (Bertin *et al* 1972). The disparity is higher still if we consider the showers to be due to all heavy primaries like iron.

In table 4, the experimental values on the charge to neutral ratio (C/N) of hadrons for two different size groups (ie $< 3 \times 10^5$ and $\geqslant 3 \times 10^5$ particles) have been compared with the calculated values for proton showers of size of the order of 3×10^5 particles. It is seen that unless $N\bar{N}$ production is considered, the disparity between the experimental and calculated values is by more than a factor of twenty. We shall consider in detail later in § 4 how the presence of heavy primaries will influence the C/N ratio.

Greider (1971) has recently carried out quite extensive Monte Carlo simulations of air showers and has considered different types of isobar and fireball production, and also $N\bar{N}$ production, and distribution of transverse and longitudinal momenta of secondaries. The results, however, are not directly available for our altitude of 2·2 km and shower sizes are not calculated, and therefore direct comparison with experimental results presents some difficulty. However, by a reasonable interpolation we have compared our results with his calculations. In Greider's calculations except in some extreme models, in general the slope of the integral energy spectrum in the range 25–800 GeV is less than 1·5. The absolute number of high energy hadrons ($\gtrsim 100$ GeV) in most of his models is higher than what is experimentally observed. The C/N ratio in all his models has a value greater than ten at a few hundred GeV and the ratio increases

Table 1. Details of models used by Murthy *et al* for simulation of hadron cascades

	Model QLN		Model LLN		Model HLN		Model IBN		
							Nucleon		
	Nucleon	Pion	Nucleon	Pion	Nucleon	Pion	Fireball	Isobar	Pion
Multiplicity, M	$2{\cdot}7\,E^{1/4}$		$5{\cdot}26 \ln (E/18+1)$		$0{\cdot}96\,E^{1/2}$		$0{\cdot}25\,E^{1/2}$	3	$0{\cdot}96\,E^{1/2}$
Inelasticity, K	0·5	1	0·5	1	0·5	1	0·2	distribution with average 0·3	1
Mean free path in g cm^{-2} air	80	120	80	120	80	120	80		120
Fraction of N$\overline{\text{N}}$ produced, f	$7(500/E+1)^{-1}$		$7(500/E+1)^{-1}$		$7(500/E+1)^{-1}$		isobar excitation probability is 0·7 $7(500/E+1)^{-1}$		$7(500/E+1)^{-1}$
Energy spectrum of created particles	exponential		exponential		exponential		distribution decided by kinematics		exponential

E is the projectile energy.

The average energy of created pions and nucleons is assumed to be proportional to their respective masses.

QL, LL, HL, and IB models are identical with QLN, LLN, HLN, and IBN models, respectively, except that all the created particles are assumed to be pions, ie $f = 0$. QL, LL, and HL stand for 'quarter law', 'log law' and 'half-law' of multiplicity variation.

Table 2. Slope of the hadron spectrum 50–800 GeV at 800 g cm^{-2}

		Calculated			
		Proton primaries		Iron primaries	
Shower size	Observed	QLN, LLN, IBN	HLN	QLN, LLN, IBN	HLN
10^5	1·2†–1·4	1·2	1·5	1·2	1·5
4×10^5	1·9†–2·3	1·2	1·5	1·2	1·5
10^6	1·9–2·3	1·2	1·5	1·2	1·5
3×10^6	1·9–2·3	1·2	1·5	1·2	1·5

† The values 1·2 and 1·9 correspond to the values of the slope when the maximum probable systematic underestimate of energy is taken into account.

Table 3. Variation of the number of hadrons ($\geqslant$ 100 GeV) with shower size

	Observed number of hadrons of $E \geqslant 100$ GeV per shower	Calculated			
		Proton primaries		Iron primaries	
Shower size		Without $N\bar{N}$	With $N\bar{N}$	Without $N\bar{N}$	With $N\bar{N}$
10^5	7·3	30	85		
4×10^5	20	120	320		
10^6	37	300	800	550	1200
3×10^6	82	800	2000		

Table 4. Charge to neutral ratio of hadrons

Energy (GeV)	Observed shower size at 800 g cm^{-2}		Calculated	
	$< 3 \times 10^5$	$\geqslant 3 \times 10^5$	Without $N\bar{N}$	With $N\bar{N}$
25	6·2±1·3	3·0±0·3	200	2–4
50	3·8±0·9	2·6±0·3	130	1·5–3
100	1·9±0·6	1·8±0·4	65	1·2–2·2
200	1·2±0·8	1·2±0·8	25	1·1–1·9

with hadron energy which is opposite to the trend seen in the present experiment. Also, in none of the models is there a trend for the energy spectrum to steepen with increasing primary energy.

Thus we come to the conclusion that the experimental results on the variation of slope of the hadron spectrum, the absolute number and charge to neutral ratio of hadrons as a function of hadron energy and shower size are not in conformity with the various Monte Carlo simulations. This suggests that either the collision characteristics at ultra-high energies are considerably different from the extrapolations from machine energies, or the primary composition is radically changing in the energy range 10^{14}–10^{16} eV, or both causes are operative. We shall now examine which of these possibilities is indicated by the properties of high energy hadrons, especially their size dependence.

3. Variation of the number of hadrons with size

The results on the variation of the number of hadrons with shower size have shown that the slope of the N_n against N_e curve flattens with increasing hadron energy. For hadrons of higher energy this flattening could even be due to a radical change of slope of the curve from a low to a high value within the shower size investigated (ie 5×10^4–5×10^6 particles at 800 g cm^{-2}). Several experiments on the variation of the number of low energy hadrons (few GeV) and of low energy muons (few GeV) with shower size have clearly shown the existence of a change of slope in this size range. The early results of Nikolsky (1956) on this change of slope were interpreted by Peters (1960) on the basis of a model of changing primary composition, which he attributed to a rigidity cut-off in the galactic magnetic field. Later, Chatterjee (1964) explained a variety of results in EAS on the basis of such a model. The behaviour of the high energy γ ray spectrum in the atmosphere (Yash Pal and Tandon 1966) and the slow increase of high energy muons with shower size (Sivaprasad 1972) also lend support to the hypothesis of a changing primary composition in the energy range 10^{14}–10^{16} eV.

Let us examine in a qualitative manner what should be the trend in the change of the average mass of the primary, if the change of slope is to be attributed to change of composition. Let $N_n(\mathrm{P})$ be the number of hadrons of energy greater than E in showers initiated by protons; $N_n(A)$ the number of hadrons of energy greater than E in showers initiated by heavy primaries of atomic mass A. Assume $N_n(\mathrm{P}) \sim N_e^{\alpha}$ for *all* sizes, then $N_n(A) \sim N_e^{\alpha} A^{1-\alpha}$. Thus the slope of the N_n–N_e curve of a heavy primary A (as also mixed composition which does not change with size) is the same as that of a proton primary. Suppose the composition undergoes a change in a size interval $N_{e(1)}$–$N_{e(2)}$. Assume that the variation of the mean atomic mass $\langle A \rangle$ in this size interval is expressed by

$$\langle A \rangle \sim N_e^{q},$$

then†

$$N_n(A) \sim N_e^{\alpha+q-\alpha q}.$$

Depending upon the values of α and q, there are four possible behaviours of a change in the slope of the N_n against N_e curve in the size interval $N_{e(1)}$–$N_{e(2)}$. These possibilities are illustrated in figure 1.

If the size $N_{e(2)}$ is identified with the value of the order of 3×10^5 as is expected from other results, it is clear that in order to explain the break the appropriate parameters are either $\alpha < 1$, $q < 0$ or $\alpha > 1$, $q > 0$. The second possibility is unlikely as α is less than 1 in the present experiment for high energy hadrons as well as in the other experimental results on the size variation of low energy hadrons and muons. Since this implies that $q < 0$, a *changing composition which becomes progressively lighter with*

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† Actually the N_n–N_e curve for a mixed composition is given by $N_n(\langle A \rangle) \sim N_e^{\alpha} \Sigma W_i A_i^{1-\alpha}$ subject to $\Sigma W_i = 1$ and $\langle A \rangle = \Sigma W_i A_i$ where the fraction of primaries of atomic mass A_i out of all primaries is W_i. To explain the effects quantitatively the actual mixed composition is replaced by $\langle A \rangle$ which is not strictly correct. The conclusions, however, are not affected by such a procedure.

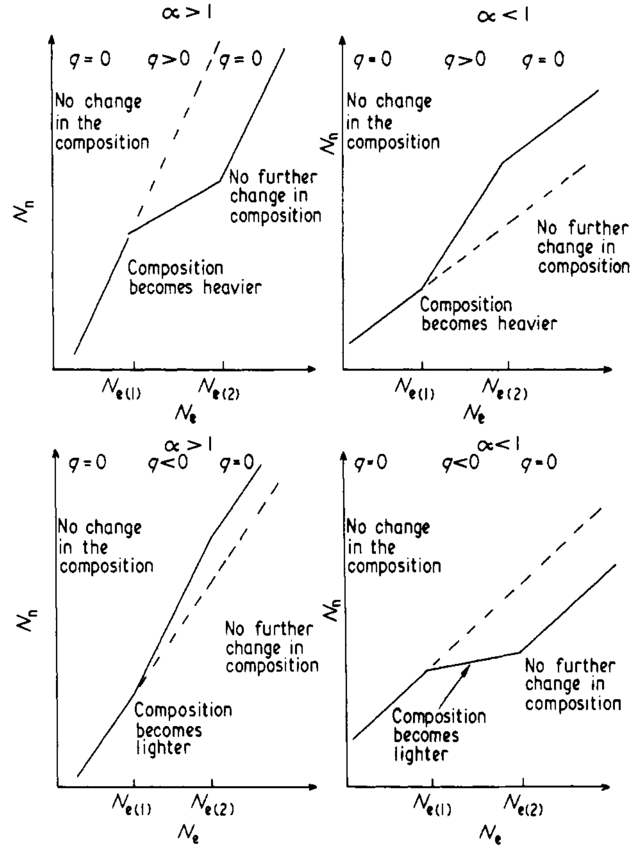

Figure 1. Sketches illustrating the behaviour of the N_n against N_e curve when the primary composition changes corresponding to the size interval $N_{e(1)}$ to $N_{e(2)}$ for different ranges of values of α and q.

increasing size is required to explain the observed changes in slope. An anomalous feature is the absence of a change of slope in the intermediate energy range of 25–100 GeV in the experimental results.

4. Charged to neutral ratio of hadrons

The ratio of charged to neutral hadrons in an EAS may be written as

$$R(E_0) = \frac{n_{\pi^\pm}(E_0) + n_{K^\pm}(E_0) + \frac{1}{2} n_{N\bar{N}}(E_0) + \frac{1}{2}}{\frac{1}{2} n_{N\bar{N}}(E_0) + n_{K^0\bar{K}^0}(E_0) + \frac{1}{2}} \tag{1}$$

where n_X is the number of hadrons of type X at the observation level produced in an EAS initiated by a nucleon of energy E_0. Equation (1) is valid even if the charge exchange probability in nucleon interactions is small ($\lesssim 20\,\%$) since, on the average, the survivor undergoes a number of interactions so that whatever be the nature of the primary nucleon the probability of the survivor being a proton is $\frac{1}{2}$.

If no $N\bar{N}$ or kaon production is considered then

$$R(E_0) = 2n_{\pi^\pm}(E_0)+1 \tag{2}$$

which increases with E_0 or size.

In the case of a shower initiated by a heavy primary of total energy E_0 and mass number A, the charge to neutral ratio is a function of energy per nucleon and not total energy, that is,

$$R(E_0)_{\text{heavy primary}} = R\left(\frac{E_0}{A}\right)_{\text{nucleon}}.$$

The experimental results indicate a decrease in the ratio with increasing size and to explain this on the basis of a change in primary composition E_0/A *must decrease with increasing* E_0. Thus the mechanism of enriching heavy primaries with increasing energy must be faster than that expected from a model invoking magnetic rigidity cut-off in which the average energy per nucleon does not decrease with increasing energy.

Assuming $n_{K^\pm} = n_{K^0\bar{K}^0}$, the fraction of nucleons and kaons among the hadrons is

$$f_{NK} = \frac{2}{R(E_0)+1}.$$

At a shower size of 3×10^6, $R = 3{\cdot}2$ so that $f_{NK} = 0{\cdot}476$. Since the total number of hadrons of energy greater than 25 GeV at size 3×10^6 is 1000, the number of nucleons and kaons is 476. Even if we assume that the showers are all due to iron nuclei the maximum contribution from survivors is 56 nucleons so that at least 420 $N\bar{N}$ pairs and kaons must be secondaries. Thus a change in composition alone cannot explain the C/N ratio at large sizes. The results on the time structure of hadrons in EAS (Tonwar and Sreekantan 1971) can be interpreted only in terms of an increase in $N\bar{N}$ production and cannot be explained by an increase in kaon production.

If the decrease in R at a shower size greater than 3×10^5 particles is to be attributed to a further increase in $N\bar{N}$ production it is obvious that the excess nucleons must arise from interactions of energies of 10^6 GeV or more near the top of the atmosphere.

5. Variation of the slope of the hadron spectrum

The slope of the integral hadron energy spectrum is obviously a function of energy per nucleon and not of total energy of the primary. Consequently in a simple superposition model the energy spectrum of high energy hadrons in a shower due to a heavy primary will be steeper than in a proton shower of the same primary energy. The experimental data show a steepening of the slope of the hadron spectrum with increase of shower size up to a size value of the order of 4×10^5. If this feature has to be explained by change of primary composition, then the transformation to heavy primaries must occur very fast so that the energy per nucleon of the primary is brought down rapidly. It may be pointed out that in a rigidity cut-off model while the primary composition is changing the average energy per nucleon does not decrease, and therefore we should not expect a steepening of slope with increasing size.

It is possible that due to the onset of collective interactions at high energies in the collisions of heavy primaries the spectrum of high energy hadrons in heavy primary showers may be considerably different from what is expected on the basis of a simple superposition model. It may be mentioned however that Orford and Turver (1969) have considered the effects of coherent interactions of heavy primaries in connection with their calculations on the behaviour of the high energy muon component and have come to the conclusion that collective interactions are of little significance as far as high energy muons are concerned.

6. Tendency for flattening of the hadron lateral distribution

As discussed in paper I (Vatcha and Sreekantan 1973), the cloud chamber results also show a tendency for the lateral distribution of high energy hadrons to flatten with increasing size, though it is not so pronounced as in the experiments with non-visual detectors. The Monte Carlo simulations indicate if at all an opposite trend—the distribution is slightly flatter in small size showers. The lateral distribution of hadrons is a function of the energy per nucleon and not the total energy of the primary. If a change in the primary composition is responsible for the flattening, then it is evident that the energy per nucleon has to be reduced rapidly with increasing size, which means that the primary composition must become rapidly heavier. The Monte Carlo simulations of Greider (1970) show that in some models an increase in the average height of first interaction of the primary tends to flatten the lateral distribution of hadrons. As the primaries become heavier the average height of primary collision increases and would thus serve as an additional cause for flattening as the primary composition becomes heavier. The mechanism of fragmentation of heavy nuclei plays an important role in the determination of the lateral distribution of hadrons. For example, if the heavy nucleus fragments in such a way that only one nucleon interacts at a time, then the effective height of the cascade maximum is lowered much more than in the case of a single act of fragmentation. Thus the amount of flattening would depend on the exact mechanism of fragmentation. What is clear is that a change from a lighter to a heavier component is necessary for the flattening to occur with increasing size.

7. The fractional hadron energy spectrum

In figure 2 we have reproduced the fractional hadron energy spectrum given in figure 8 of the previous paper, wherein the observed integral hadron energy spectrum is plotted as a fraction of the primary energy of the associated shower after classifying the data into two broad groups $N_e < 3 \times 10^5$ and $N_e \geqslant 3 \times 10^5$. In general, these hadrons at the observational level are a mixture of the survivors of the incident primaries and of secondaries produced in various collisions. In figure 2 the different curves correspond to the spectra of the surviving primaries calculated for primaries of mass numbers 1, 4, 16 and 64 (chosen for convenience) and on the basis of certain pairs of values of elasticity and interaction mean free path. The broken curve 'C' represents a mixed composition approximating the one known at lower energies. The calculated spectra† are obtained from the fluctuations in the number of interactions of the primary before reaching the level of observation.

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† These curves were obtained by Dr B K Chatterjee.

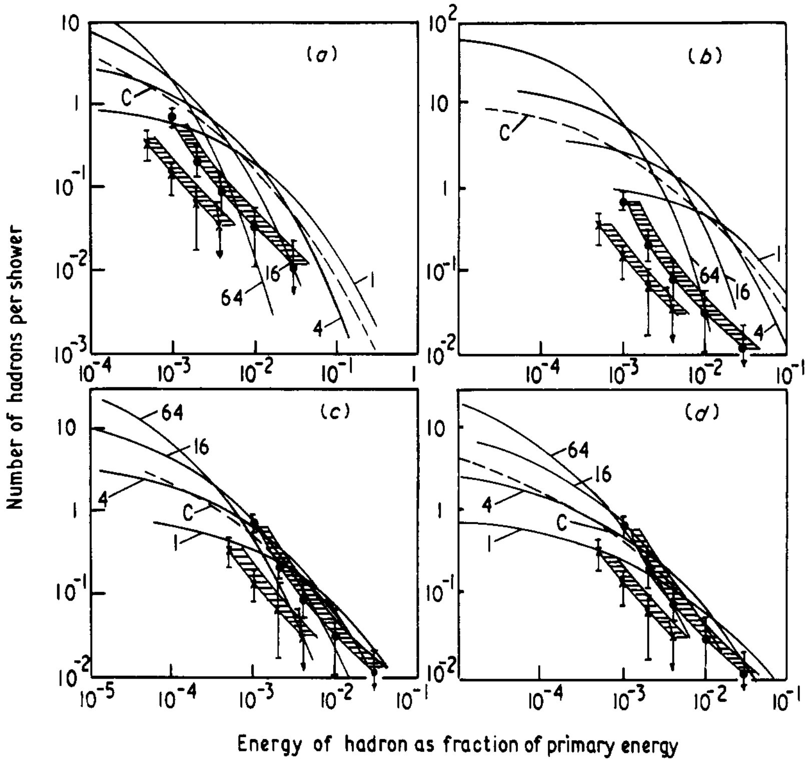

Figure 2. The fractional integral energy spectrum of hadrons of energy greater than 200 GeV for the size groups less than $3{\cdot}2 \times 10^5$ (●) and greater than $3{\cdot}2 \times 10^5$ (✕) at Ooty. The hadron energy is expressed as a fraction of the primary energy of the associated EAS. The experimental points are compared with calculated integral energy spectra for different pairs of values of interaction mean free path λ and elasticity ϵ. The different curves in each figure correspond to different mass numbers (1, 4, 16, 64) of primaries. The dashed curve C corresponds to a mixed composition. (*a*) $\lambda = 80\,\mathrm{g\,cm^{-2}}$, $\epsilon = 0{\cdot}5$; (*b*) $\lambda = 80\,\mathrm{g\,cm^{-2}}$, $\epsilon = 0{\cdot}6$; (*c*) $\lambda = 67\,\mathrm{g\,cm^{-2}}$, $\epsilon = 0{\cdot}5$; (*d*) $\lambda = 80\,\mathrm{g\,cm^{-2}}$, $\epsilon = 0{\cdot}4$.

For a primary of atomic mass 'A' the number of nucleon survivors at the level of observation with a fractional energy greater than K is given by

$$N_{\mathrm{n}}(>K) = A\sum_{l=0}^{R}\frac{\mathrm{e}^{-X}X^{l}}{l!} \qquad \text{with} \qquad K = \frac{\epsilon^{R}}{A},$$

where $X = 800/\lambda$, $800\,\mathrm{g\,cm^{-2}}$ being the atmospheric depth corresponding to Ooty, λ = interaction mean free path, and ϵ = elasticity in nucleon–air nucleus collisions. Since the spectrum of the survivors of primaries of a given A is determined only by the value of λ and ϵ for any given depth X, and if λ, ϵ and A do not change with energy, then the spectrum should remain the same for all sizes. If the comparison between calculation and observation is to be meaningful, survivors must be well separated from secondaries so that secondaries should be negligible in the energy region considered here.

If we consider the survivor spectra for $\lambda = 80\,\mathrm{g\,cm^{-2}}$ and $\epsilon = 0{\cdot}5$, which are the normally accepted values at lower energies, then the observed spectra are well below the expected survivor spectra for protons as well as for heavier primaries, and there is no scope for any contribution from secondaries at all. The disagreement is more serious in the case of fractional hadron energy spectra for the large size ($> 3 \times 10^5$) showers. As already pointed out there should be no size dependence of this spectrum for fixed λ, ϵ and A whereas the experimental results clearly show a difference in the spectra for showers of size less than 3×10^5 and greater than 3×10^5.

If we consider the pair of values $\epsilon = 0{\cdot}4$ and $\lambda = 80$ or $\epsilon = 0{\cdot}5$ and $\lambda = 68$, then there is better agreement for showers of size less than 3×10^5, but the same values do not give a spectrum that is compatible with the observed spectrum for larger sizes. It is also clear that a change in composition at a size value of about 3×10^5 does not resolve the discrepancy. A more drastic change in the value of ϵ and/or λ is necessary for the larger sizes. Therefore one feature that seems to emerge is that the values of ϵ and/or λ are not the same at ultra-high energies.

It must be emphasized that the comparisons indicate only trends and the actual discrepancy is expected to be larger for the following reasons:

(i) Statistical errors in the energy estimate of hadrons tend to increase the observed number of hadrons at any energy.

(ii) Size to primary energy conversion is done on the basis of proton primaries only so that E_0 may be underestimated and hence 'K' may be overestimated.

(iii) Secondaries would contribute at least for low values of 'K'. Thus normally the contamination from secondaries would tend to steepen the spectrum of survivors unless the energy spectrum of secondaries is similar to those of survivors. Possible sources of underestimating either the flux or the energy of hadrons in the experimental results have been considered in paper I and it appears unlikely that any systematic biases could affect the conclusions. Thus a change in the characteristics of strong interactions from a few TeV to several hundred TeV is most likely and a change in primary composition alone cannot explain the above results.

The present results agree qualitatively with those of Grigorov *et al* (1965), who have concluded that at sizes of 10^5 particles or more at about 3·2 km above sea level, the concept of a leading particle propagating the air shower should probably be abolished.

8. Conclusions

The steepening of the hadron spectrum, the variation of the C/N ratio, the tendency for flattening of the lateral distribution with increasing size, all suggest that if the size dependence is due to a change of primary composition, then the composition should change rather suddenly from a lighter to a heavier component with increasing primary energy from 10^{14} to 10^{16} eV. On the other hand, the variation of the number of low and high energy hadrons and muons requires a change of primary composition from a heavier to a lighter component with increasing primary energy. To explain the variation of the average steepness parameter of electrons with size as well as to understand the correlations between the structure functions of hadronic, muonic and electronic components observed at Ooty, Chatterjee (1964) suggested a model in which the primary composition becomes increasingly lighter as the size increases. Thompson *et al* (1969) have shown that several results on the muonic component of EAS either require no change in primary composition or a change of composition which becomes progressively lighter with increasing primary energy at energies around 10^{15} eV.

Thus the hypothesis of changing primary composition leads to serious contradictions. Moreover, the rather low value of C/N ratio for large size showers where the absolute number of hadrons is quite high indicates the need for copious production of nucleon–

antinucleon pairs. The further decrease in the C/N ratio at higher shower sizes implies a further increase in the production of $N\overline{N}$'s, which has to necessarily take place only at energies of 10^{15} eV or more. The distinct difference in the fractional hadron energy spectra for different size groups and the observation that the experimental spectra fall below those expected for survivors on the basis of normally accepted values of inelasticity and interaction mean free paths, suggest that these parameters also need revision at higher energies.

In a subsequent paper we present trends of ultra-high energy interactions suggested by the change in the properties of high energy hadrons in air showers and discuss to what extent such trends succeed in explaining some of the important properties of air showers.

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